

NAG C Library Function Document

nag_deviates_studentized_range (g01fmc)

1 Purpose

nag_deviates_studentized_range (g01fmc) returns the deviate associated with the lower tail probability of the distribution of the Studentized range statistic.

2 Specification

```
double nag_deviates_studentized_range (double p, double v, Integer ir,
NagError *fail)
```

3 Description

The externally Studentized range, q , for a sample, x_1, x_2, \dots, x_r , is defined as

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i . The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \dots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, \bar{T}_{largest} and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error} , over the number of observations in each group, n , i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups, can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left(r \int_{-\infty}^\infty \phi(y) (\Phi(y) - \Phi(y - qx))^{r-1} dy \right) dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^y \phi(t) dt.$$

For a given probability p_0 , the deviate q_0 is found as the solution to the equation

$$P(q_0; v, r) = p_0, \tag{1}$$

using a root finding procedure. Initial estimates are found using the approximation given in Lund and Lund (1983) and a simple search procedure.

4 References

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32** (2) 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

1:	p – double	<i>Input</i>
	<i>On entry</i> : the lower tail probability for the Studentized range statistic, p_0 .	
	<i>Constraint</i> : $0.0 < p < 1.0$.	
2:	v – double	<i>Input</i>
	<i>On entry</i> : the number of degrees of freedom, v .	
	<i>Constraint</i> : $v \geq 1.0$.	
3:	ir – Integer	<i>Input</i>
	<i>On entry</i> : the number of groups, r .	
	<i>Constraint</i> : $ir \geq 2$.	
4:	fail – NagError *	<i>Input/Output</i>
	The NAG error parameter (see the Essential Introduction).	

6 Error Indicators and Warnings

NE_INT

On entry, **ir** = $\langle value \rangle$.
Constraint: $ir \geq 2$.

NE_ACCURACY

Warning - There is some doubt as to whether full accuracy has been achieved.

NE_INIT_ESTIMATE

Unable to find initial estimate.

NE_REAL

On entry, $p \leq 0.0$ or $p \geq 1.0$: **p** = $\langle value \rangle$.
On entry, **v** = $\langle value \rangle$.
Constraint: $v \geq 1.0$.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The returned solution, q_* , to equation (1) is determined so that at least one of the following criteria apply.

- (a) $|P(q_*; v, r) - p_0| \leq 0.000005$
- (b) $|q_0 - q_*| \leq 0.000005 \times \max(1.0, |q_*|)$.

8 Further Comments

To obtain the factors for Duncan's multiple-range test, equation (1) has to be solved for p_1 , where $p_1 = p_0^{r-1}$, so on input **p** should be set to p_0^{r-1} .

9 Example

Three values of p , ν and r are read in and the Studentized range deviates or quantiles are computed and printed.

9.1 Program Text

```
/* nag_deviates_studentized_range (g01fmc) Example Program.
*
* Copyright 2001 Numerical Algorithms Group.
*
* Mark 7, 2001.
*/
#include <stdio.h>
#include <nag.h>
#include <nag_stdlb.h>
#include <nagg01.h>

int main(void)
{
    /* Scalars */
    double p, v, valq;
    Integer exit_status, i__, ir;
    NagError fail;

    exit_status = 0;

    INIT_FAIL(fail);
    Vprintf("g01fmc Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[^\n] ");

    Vprintf("\n%s\n\n", " p      v      ir      Quantile ");
    for (i__ = 1; i__ <= 3; ++i__)
    {
        Vscanf("%lf%lf%ld*[^\n] ", &p, &v, &ir);

        valq = g01fmc(p, v, ir, &fail);
        if (fail.code == NE_NOERROR || fail.code == NE_ACCURACY)
        {
            Vprintf("%5.2f%2s%4.1f%ls%3ld%ls%10.4f\n", p, "", v, "", ir,
", valq);
        }
        else
        {
            Vprintf("Error from g01fmc.\n%s\n", fail.message);
            exit_status = 1;
            goto END;
        }
    }

    END:
    return exit_status;
}
```

9.2 Program Data

```
g01fmc Example Program Data
0.95 10.0 5
0.3 60.0 12
0.9 5.0 4
```

9.3 Program Results

```
g01fmc Example Program Results
```

p	v	ir	Quantile
0.95	10.0	5	4.6543
0.30	60.0	12	2.8099
0.90	5.0	4	4.2636
